LOW FREQUENCY INVESTIGATION OF THE ION-ION HYBRID RESONANCE IN TWO-ION SPECIES PLASMAS

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by

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ABSTRACT

The ion-ion hybrid resonance was investigated in neon-krypton and neon-argon plasmas. A signal was transmitted from one coil to a second coil to which the first was inductively coupled. A strong noise interference was encountered. A lock-in amplifier and filter system was tested for noise rejection, as was a system employing a third coil and a differential amplifier circuit. Variation of power transmitted and varying coil spacing were also tried. Some results were obtained but they were not definite or reproducible since the noise rejection techniques were not adequate. A review of the various theories pertaining to the ion-ion hybrid resonance and recommendations for further work are presented.

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TABLE OF SYMBOLS

Π = plasma frequency

Ω_i = ion cyclotron frequency of i^{th} ion

Ω_e = electron cyclotron frequency

ω_{ii} = ion-ion hybrid frequency at $\theta = 90°$

ω_{ei} = electron-ion hybrid frequency at $\theta = 90°$

θ' = direction of wave propagation relative to magnetic field at resonance

X_i = relative concentration of i^{th} ion

f_i = ratio of electron mass to i^{th} ion mass

1. Introduction.

This thesis is an investigation of a means of detecting the ion-ion hybrid resonance. This resonance occurs when an electromagnetic wave is propagated at the resonance frequency through the plasma which contains positive ions of more than one species and is confined in an axial magnetic field. The ion-ion hybrid resonance frequency is a function of the magnetic field, the masses of the ion species, and their relative concentrations. The resonance frequency lies between the ion cyclotron frequencies of the individual ion components.

This resonance frequency is of interest since a means for its detection would provide a valuable diagnostic tool for multi-component plasmas, as found, for example in the atmosphere. If the ion species and magnetic field were known, it would be possible to determine the relative proportions of the ion species in the plasma with detection of the ion-ion hybrid resonance frequency.

The resonance is also of interest since at this resonance the ions are heated preferentially over the electrons. The ions absorb energy from the wave and therefore high ion temperature could result. This is of great importance in development of a controlled thermonuclear fusion reactor for power generation.

Our diagnostic probe consisted of two coaxially mounted coils with one inductively coupled to the other. The probe was immersed in a two ion species plasma generated by a reflex arc and the probe was oriented perpendicularly to the axial magnetic field. The signal transmitted was swept from below the lower ion cyclotron frequency to above the upper ion cyclotron frequency and the resonance of the signal induced on the second coil was observed.

This project was proposed by Mr. W. P. Jones, Research Scientist of NASA, Ames Research Center, Moffett Field, Sunnyvale, California. Mr. Jones also provided some needed materials and equipment as well as guidance in this research.

2. Theory.

Buchsbaum [1] considered a plasma consisting of two ion species and derived the propagation constant, k_x, for this system from [2]

$$\frac{1}{2} k_x^2 = \frac{k_r^2 k_\ell^2}{k_r^2 + k_\ell^2}$$

He assumed a collisionless plasma and he further defined k_ℓ and k_r as the propagation constants of left and right circularly polarized waves which were propagating with the static magnetic field. Then after some simplification, he arrived at

$$\frac{k_x^2}{k_o^2} = 1 + \pi^2 \Big\{ \pi^2 \left[\omega^2 - (X_2\Omega_1 + X_1\Omega_2)^2\right] - \left[\omega^2 - \Omega_e(X_1\Omega_1 + X_2\Omega_2)\right] \left[\omega^2 - \frac{X_2f_1 + X_1f_2}{X_1f_1 + X_2f_2}\Omega_1\Omega_2\right] \Big\} \div \Big\{ (\omega^2 - \Omega_e^2)(\omega^2 - \Omega_1^2)(\omega^2 - \Omega_2^2) - \pi^2\left[\omega^2 - \Omega_e(X_1\Omega_1 + X_2\Omega_2)\right]\left[\omega^2 - \frac{X_2f_1 + X_1f_2}{X_1f_1 + X_2f_2}\Omega_1\Omega_2\right] \Big\} .$$

From this it can be seen that for larger plasma densities three resonance conditions exist,

(1) $\omega \simeq \pi$,

(2) $\omega_{ei}^2 \simeq \Omega_e \left(X_1\Omega_1 + X_2\Omega_2 \right)$,

(3) $\omega_{ii}^2 = \dfrac{X_2f_1 + X_1f_2}{X_1f_1 + X_2f_2}\Omega_1 \Omega_2$.

The first two involve both electrons and ions while the third involves only the ions. The third is referred to as the ion-ion hybrid resonance and is the one of concern to us.

Therefore, we see that the ion-ion hybrid resonance frequency is a function of the relative concentrations of the two ion species, the masses of the species and the magnetic field to which this plasma is subjected.

Buchsbaum [3] also states that as the relative concentration of the heavier ion is increased, the ion-ion hybrid resonance frequency approaches the cyclotron frequency of the lighter ion. (See Appendix I.)

It has been shown [1,4] that the electron velocity is much less than the ion velocities at the resonance condition. Further, the ion species oscillate out of phase with each other. At this resonance, then, it should be possible to heat the ions preferentially over the electrons since the ratio of the drift energies, U_+/U_- , is of the order of M_+/M_- or at least 2000 to 1. This increase of energy in the ions will be the result of an increased absorption of the transmitted power.

The amplitude of the resonance is a function of the relative concentration and there is an optimum concentration for each combination of ion species which is a function of the ratio of the masses of the two species [1]. Using the optimum concentration gives a resonance of maximum amplitude. (See Fig. 1 and Fig. 2.) Also, there is a requirement that

$$\pi^2 \gg \Omega_e \Omega_i$$

for the ion-ion hybrid resonance to be reached. When

$$\pi^2 \ll \Omega_e \Omega_i$$

the ion cyclotron resonance is obtained. It has been found that an increase in density of about two orders of magnitude is necessary to shift

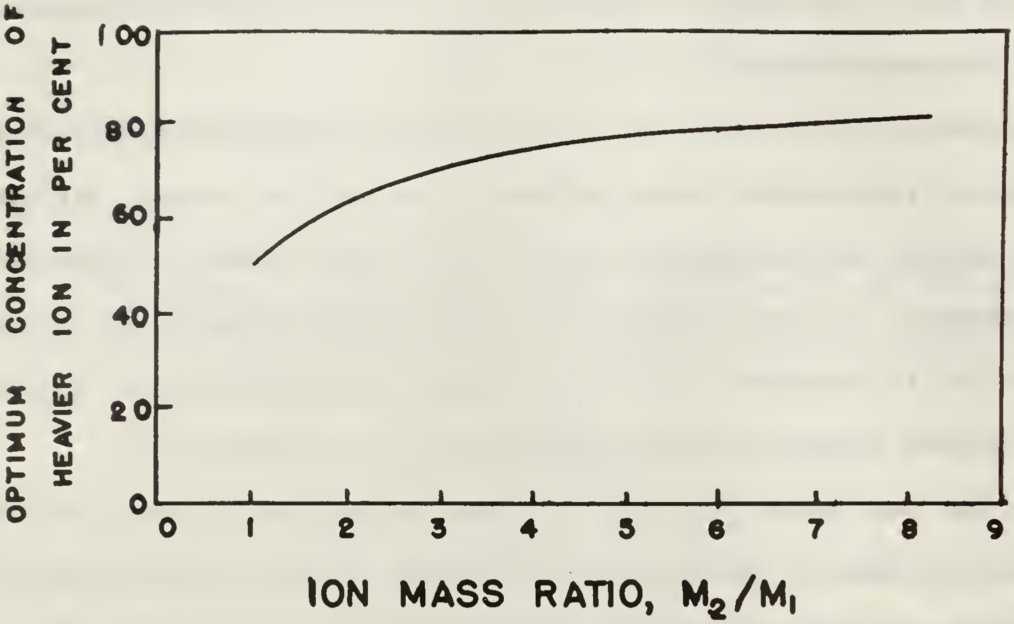


Fig. 1 Optimum concentration of the heavier ion as function of the ion mass ratio [1]

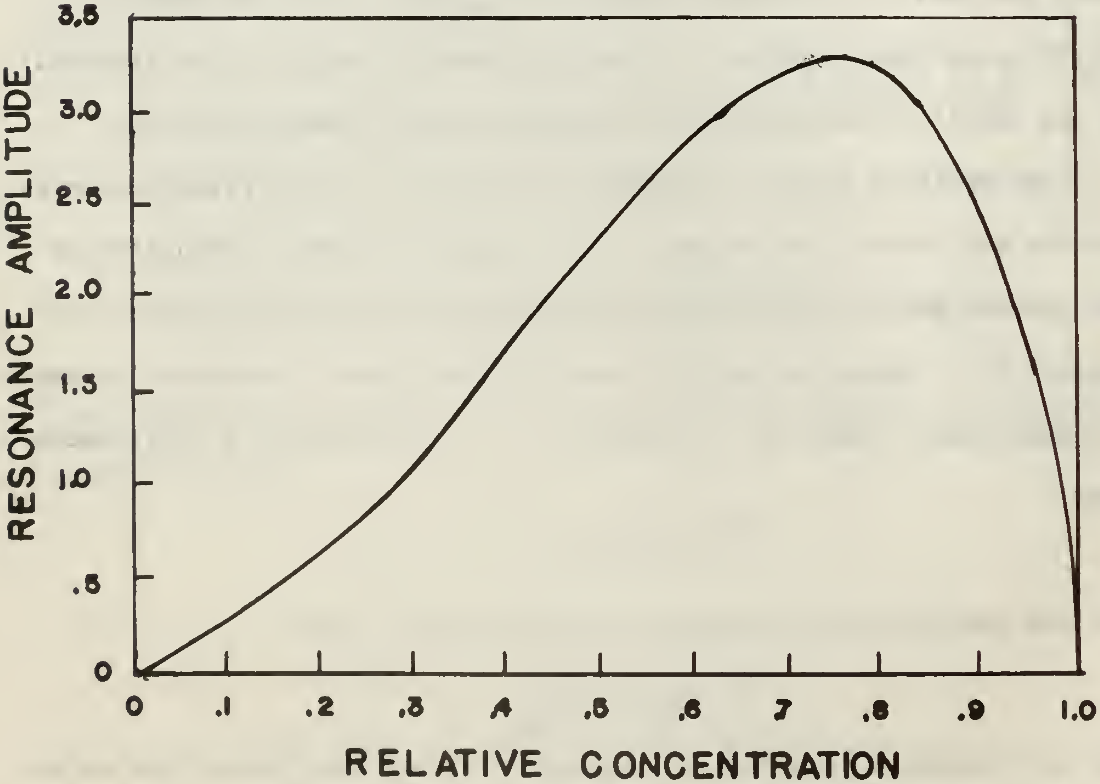


Fig. 2 Magnitude of resonance for ion mass ratio of 4. [1]

from the ion cyclotron resonance to the ion-ion hybrid resonance [3].

At the optimum concentration the greater amount of the energy absorbed goes to the lighter ion specie [1]. It can be seen that this can be shifted at will by varying the concentration from the optimum. However, as this shift is made the resonance amplitude is decreased (Fig. 2) and from this it follows that the total energy transferred to two ion species decreases.

Buchsbaum [3] performed experiments related to the ion-ion hybrid resonance in the positive column of an arc discharge using hydrogen and helium. The hydrogen ionized in the H^+ and H_2^+ forms. Magnetic fields up to 85 k gauss were employed with frequencies from 6.2 to 27.3 MHz. Measurements were made of absorption as a function of magnetic field and indications of the ion-ion hybrid resonance were obtained. A quantitative comparison with theory was not possible since the relative concentration of the ions was not known.

Yakimenko [5] independently arrived at the same equation for the propagation constant. He also concurred with the physical conclusions of Buchsbaum, e.g. the ions oscillate out of phase with each other at the ion-ion hybrid resonance.

Reshotko [6] derives a general dispersion relation for an arbitrary angle to the magnetic field. This derivation, which follows the method of Stix [7], is for a plasma consisting of n species. This equation is then limited to a plasma with two ion species and terms of the order of $\frac{M_e}{M_i}$ compared to one are dropped. The result is

$$\left[(1+\cot^2\theta)\omega^2-\pi^2\cot^2\theta\right](\omega^2-\Omega_1^2)(\omega^2-\Omega_2^2)(\omega^2-\Omega_e^2)-\omega^2\pi^2(\omega^2-\omega_{ei}^2)(\omega^2-\omega_{ii}^2)=0$$

where ω_{ei} and ω_{ii} are the electron-ion hybrid and the ion-ion hybrid resonances respectively as shown on page 10. From this equation the following general relations for hybrid frequencies (ω) are obtained,

$$\text{(a)} \quad \omega^2 = \omega_{ei}^2 \sin^2\theta + \Omega_e^2 \cos^2\theta$$

and (b)

$$\omega^2 = \frac{\Omega_1^2 + \dfrac{\omega_{ei}^2 \omega_{ii}^2}{\Omega_e^2} \tan^2\theta}{1 + \dfrac{\omega_{ei}^2}{\Omega_e^2} \tan^2\theta}$$

When $\theta = 90^\circ$ the original hybrid resonances are recovered.

From the equations (a) and (b) above it is shown [6] that for angles less than 85°, the resonances are approximately at the cyclotron frequencies of the ion species concerned. Therefore, small deviations of the direction of propagation will result in large changes in the resonant frequency. For a plasma consisting of 50% H^+ and 50% H_2^+ ions, Fig. 3 shows the resonant frequencies for high plasma density.

Reshotko further shows that the distribution of energy to the electrons and ion species for $\theta = 90^\circ$ at $\omega = \omega_{ii}$ is

$$U_e : U_1 : U_2 = \frac{1}{\Omega_e} : X_1 \Omega_1 \frac{(\omega_{ii}^2 + \Omega_1^2)}{(\Omega_1^2 - \omega_{ii}^2)} : X_2 \Omega_2 \frac{(\omega_{ii}^2 + \Omega_2^2)}{(\Omega_2^2 - \omega_{ii}^2)}$$

If the direction is changed from 90°, the energy to the electrons and the heavier ion specie is decreased while that to the lighter ion is increased. Also the resonance frequency tends to the cyclotron frequency of the lighter ion.

Swanson [8,9] has derived a dispersion relation for a two-ion species plasma contained in a cylindrical wave guide. From this relation he concludes that there is a critical density for obtaining the ion-ion hybrid resonance for a plasma in a wave guide. This critical density is

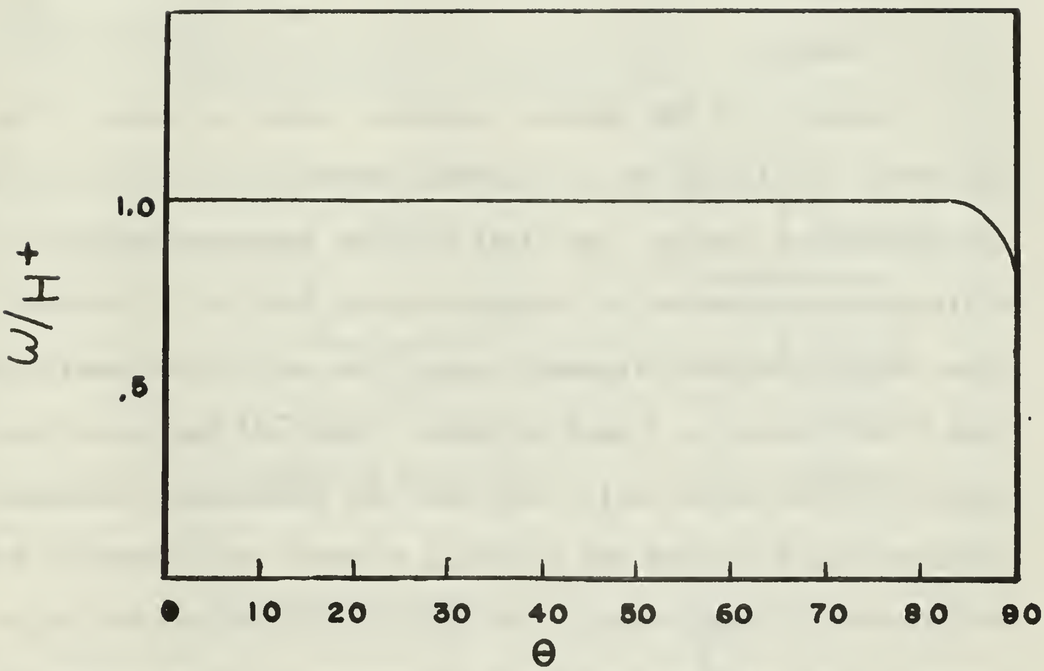


Fig. 3 Resonant frequency in a 50% H^+-50% H_2^+ plasma as a function of Θ. Frequency is normalized by the ion cyclotron frequency of H^+. [6]

a function of the masses of the two ion species, their relative concentrations and the waveguide radius. As the density is lowered toward the critical density the ion-ion hybrid resonance becomes more apparent. When density is just below the critical the effect of the hybrid resonance disappears.

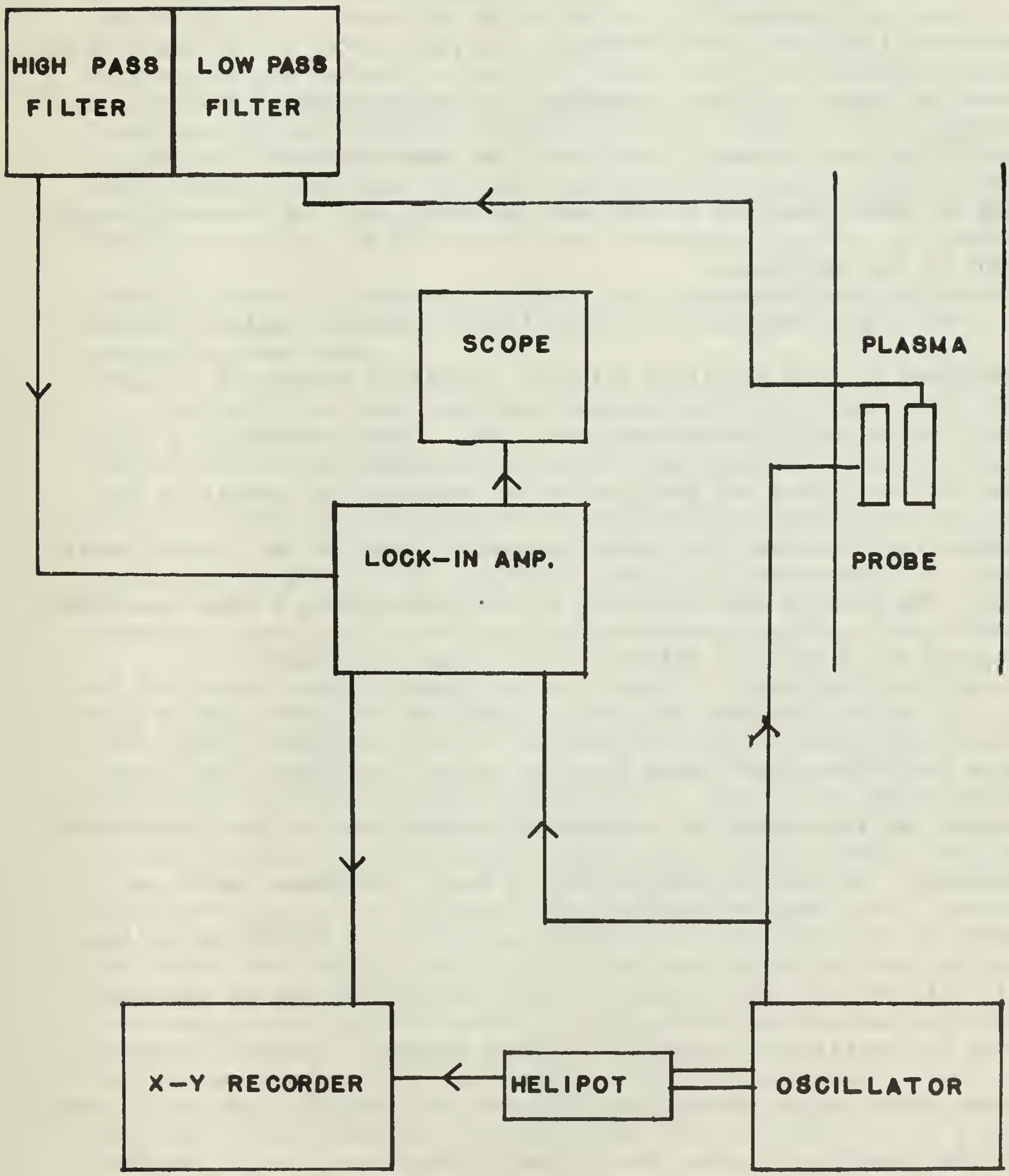
3. Equipment

a. General

A schematic of the general equipment layout is shown in Fig. 4. The probe, consisting of two coaxially mounted coils, was immersed in a two-ion species plasma. The first coil was impedance matched to an oscillator at frequencies up to approximately 100 KHz to provide a level power output for this frequency range. The oscillators used provided from 15 milliwatts to 1 watt of power. This coil was inductively coupled to the second coil. This coil was connected to a filter system to reject the noise and the resulting response was introduced into an X-Y recorder. Simultaneously the signal could be observed on an oscilloscope. The response then could be compared to the response of the receiving coil in a vacuum and any change of received signal determined.

b. Plasma Device

The plasma device used in this investigation of the ion-ion hybrid resonance was of the reflex arc type with a hollow cathode discharge. The central plasma tube is surrounded by six magnet coils coaxial with the tube. These magnets produce an axial magnetic field variable from 600 gauss to 9000 gauss which is homogeneous within $\pm$ 2.5% along the length of the column. At the time of this investigation the column was composed of pyrex sections to form a nine foot column with viewing ports and probe stations situated on either side, 14 inches center-to-center.



SCHEMATIC OF EQUIPMENT LAYOUT (FIG. 4)

In the future the pyrex column will be replaced with a stainless steel column.

c. Filter System

A filter system was necessary to reject the noise received. The received signal was passed through a low pass filter set at about 10 KHz above the upper cyclotron frequency. It was then passed through a high pass filter set at about 5 KHz below the lower cyclotron frequency. This in effect then was a pass band including only the frequency range swept by the oscillator.

The signal then was introduced into a Princeton Applied Research Laboratory Lock-in amplifier which is capable of separating a signal from a noise 40 db greater than the signal. Simultaneously, a line from the oscillator and parallel to the transmission circuit to the transmit coil carries the signal to another input of the lock-in amplifier. The lock-in amplifier then rejects noise using a phase sensitive detector and a low pass filter as its primary components.

d. An X-Y Recorder was used to trace the response. The Y input was a rectified signal taken from the lock-in amplifier. The input desired for the x-axis was a potential proportional to the transmitted frequency. In order to obtain this, a small synchronous motor was geared to the oscillator drive shaft and also to a 10,000 ohm helipot. A 1½ volt dry cell was connected across the helipot and as the motor drove the oscillator through the desired frequency ranges a potential proportional to the transmitted frequency was available for the X input. The time required to cover the frequency range was 10 to 15 seconds.

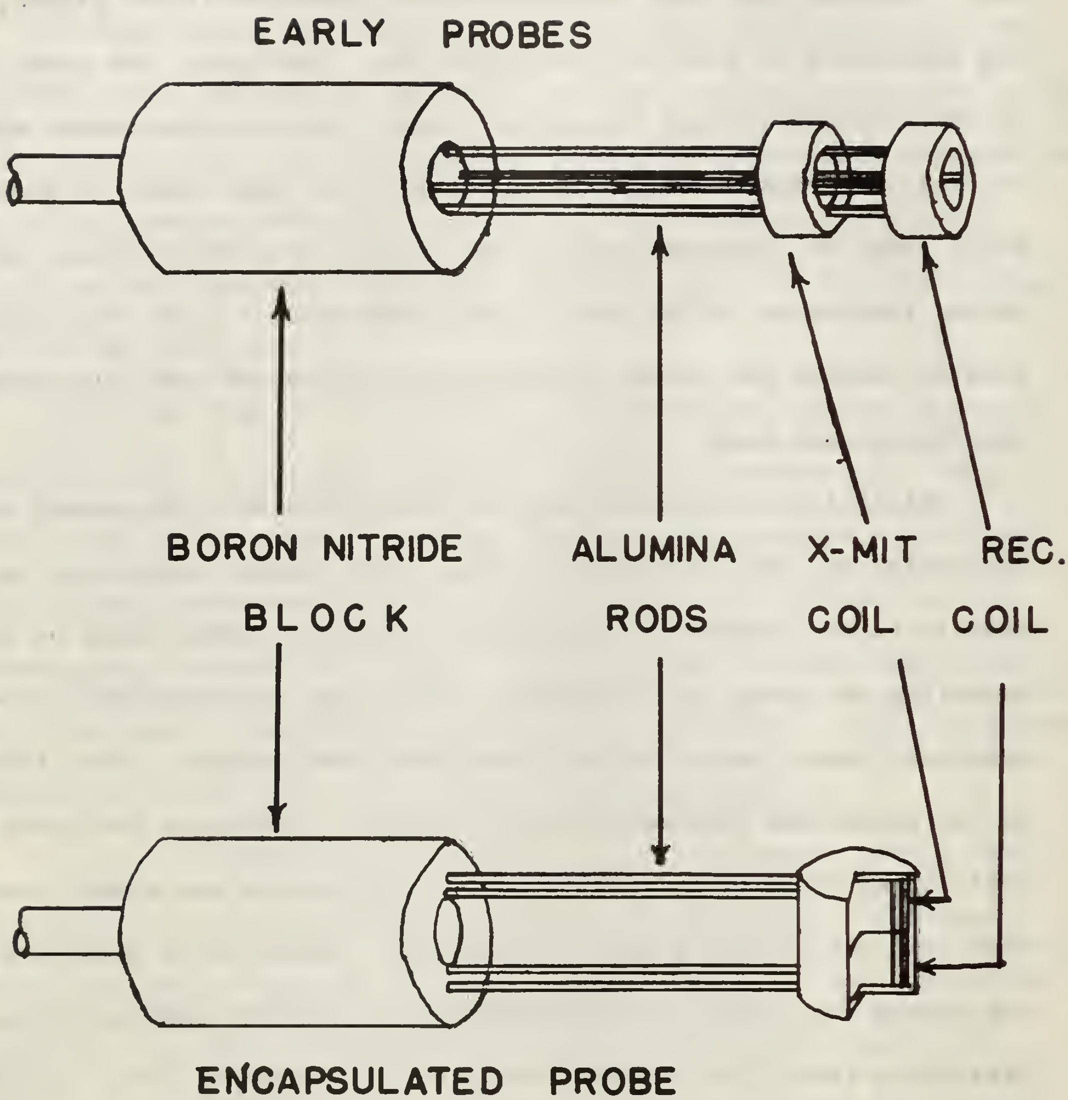
e. Probes (Fig. 5)

The heart of the probe used in these experiments was two coils which

were inductively coupled and immersed in the plasma approximately perpendicular to the magnetic field. These coils consisted of wire wound on boron nitride cores which were initially one inch in diameter. However, this was too large compared to the dimension of the plasma, causing difficulty in starting the reflex arc. Therefore, the outer diameter of the core was reduced to one-half inch. The wire was coated with Sauereisen ceramic cement. It was found that light coats of Sauereisen baked under an infrared heating lamp between successive coats, provided better insulation of the wire to the plasma heat. Also, this baking greatly reduced the amount of impurities introduced into the vacuum system during pump down.

Initially the wire used was insulated with only the normal varnish insulation and the Sauereisen coating. The varnish insulation was quickly (2 to 3 minutes) destroyed and the wire either fused or broke requiring the probe to be replaced. Next wire insulated with a ceramic material, Ceroc, was tried with the Sauereisen coating. The lifetime of the probes was increased but not greatly. Therefore the probe was built using Ceroc wire and a boron nitride capsule was placed over the wire coil and held in place by Sauereisen. These coils were used in the plasma for periods of approximately ten minutes without failure. Wire sizes from 36 to 44 were used with 100 turns per coil. Initially the probes were constructed with an open core which allowed the plasma to penetrate and provide maximum coupling with the transmitted signal. The encapsulated probe (Fig. 5) did not have this feature. It is felt that any future probe constructed should have an open core to provide this increased coupling effect with the plasma.

These coils mounted on alumina tubes were connected to a



TYPICAL PROBES

boron nitride mounting block as shown in Fig. 4. The lead wires from the coils were passed through alumina tubes and were connected via a feed-thru to the coaxial cables in the stainless steel tube. This tube entered the plasma machine port through a bellows and O-ring arrangement on a brass plate. This bellows arrangement provided 3 degrees of freedom for movement of the probe in the plasma while maintaining the vacuum. The coaxial cables were connected to a feed-thru which was silver soldered in place at the exterior end of the stainless steel tube.

The first probes constructed had the receive coil closer to the center of the column, i.e. both coils were coaxial with the stainless steel tube. This subjected the receive coil to greater heat than the transmit coil and resulted in frequent receive coil failures. The improved orientation scheme is as is shown on Fig. 4. This coil was placed in the column in a vertical position.

4. Experimental Work

It was decided to use two coils, one to transmit a signal and the other to receive it. When immersed in a plasma there is a large damping factor for the transmitted signal. Therefore, the coils were closely spaced and, in fact, were operated in the inductively coupled field. Since a relatively flat response curve was desired in order to permit detection of the ion-ion hybrid resonance, it was necessary to operate below 100 KHz. Above this frequency harmonic effects were expected for the inductively coupled coil systems [10] and were experimentally verified. The experimental verification consisted of plotting response vs. frequency using the X-Y recorder for the coil pairs. This then limited the frequency range. The minimum field available for

operation of the reflex arc was 600 gauss. For this field the cyclotron frequencies of the inert gases were approximately as follows:

Helium	240	KHz
Neon	46	KHz
Argon	24	KHz
Krypton	11	KHz

It can be seen then that neon, argon, krypton and heavier inert gases were the only ones available for use under the first condition. It was assumed that the theories of Reshotko [6] pertaining to direction of propagation did not apply to this experiment, since they were based on wave propagation theory. The resonance was expected at that frequency given by Buchsbaum's equation

$$\omega_{ii}^2 = \frac{X_2 f_1 + X_1 f_2}{X_1 f_1 + X_2 f_2} \Omega_1 \Omega_2 ,$$

which is for transverse propagation. The probes were oriented approximately perpendicular to the central axis of the plasma chamber. However, because of the assumption stated above no effort was made to insure that the probe was perpendicular to the magnetic field.

Buchsbaum [3] demonstrated that it was necessary for

$$\pi^2 \gg \Omega_e \Omega_i$$

in order to obtain the ion-ion hybrid resonance. Using the facts that

(1) $$\pi^2 = \frac{n e^2}{m_e \epsilon_o} ,$$

(2) $$\Omega_e = \frac{e B_o}{m_e} ,$$

(3) $$\Omega_i = \frac{e B_o}{M_i} ,$$

the original inequality above can be converted to the following form becoming a condition on density.

$$n \gg \frac{B_o^2 \epsilon_o}{M_i}$$

For the gases and maximum fields employed in these investigations, this inequality therefore required that the density be much greater than 10^{14} ions per cubic meter. Gall and Oleson [11] showed using Langmuir probes that for an argon plasma the ion density was on the order of 10^{18} per cubic meter. Their investigation was conducted over the same range of operating conditions and in the same machine used in the present investigation. It was therefore assumed that the ion densities in these experiments were in the neighborhood of those they obtained and therefore much greater than the required 10^{14} per cubic meter.

It was further assumed that the density profile would be similar to that obtained by Gall and Oleson [11] (Fig. 6). Therefore, the probes were placed in the region from 20 to 40 mm from the center of the beam. In this region it can be seen that the density is almost constant while the probe is still out of the hottest region of the plasma. A reason for desiring to operate in this relatively constant portion of the radial density profile is that Buchsbaum [3] states that the relative concentrations of the ion species may vary with the total density.

Initial investigations were made using a mixture of argon and neon. These gases were leaked through gas regulators and using pressure meters a mixture of 90% argon and 10% neon was roughly approximated. In order to have an accurate relative concentration of neutrals a premixed bottle of 75% krypton and 25% neon was obtained from J. T. Baker Company. It was assumed the relative concentrations of ions then would approximate this neutral mix. This particular mixture was used since

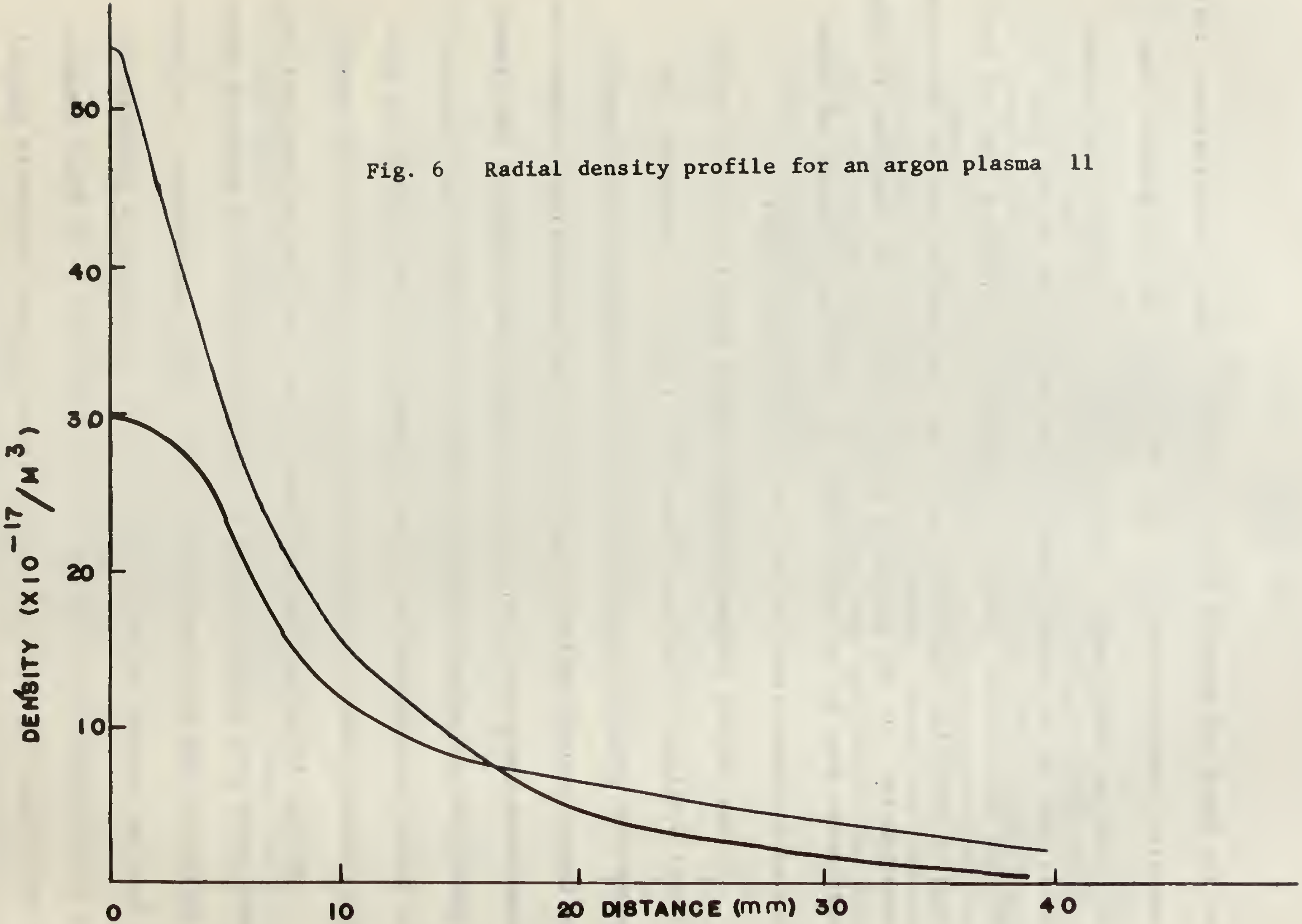


Fig. 6 Radial density profile for an argon plasma 11

Buchsbaum [1] (Fig. 1) shows that for an ion mass ratio of 4, an optimum resonance is obtained for a mixture containing 75% of the heavier ion.

Initially the probe was placed in measuring position, i.e. 20 mm from the column center, and the plasma then started. The Sauereisen coating was broken by the dynamic change in starting the reflex arc and by electron or ion bombardment. This revealed the copper wire which was then easily melted. Therefore, the probe was placed in the port arm until the steady condition was reached and then moved into the operation position. Other problems and solutions pertaining to probe construction are contained in Section 3 e.

An oscillator with a maximum power of 15 milliwatts was first used with coils spaced one centimeter apart, which is much less than one wavelength for the frequencies used. The received signal was a strong noise signal and the noise could not be rejected with the scheme in Fig. 4. Therefore in order to try to overcome the damping of the transmitted signal and increase the ratio of signal to noise an oscillator with a maximum power of one watt was used. However, it was still not possible to reject the noise to obtain a clear signal.

Therefore, a third coil was introduced into the system. The purpose of this coil was to pick up the noise signal received on the second coil and this noise signal then was introduced into a differential amplifier circuit of a Tektronix 545 oscilloscope. These two coils were 2.5 centimeters from center to center at the same distance from the column axis, i.e. one coil was 2.5 centimeters upstream in the column. Woehler [12] found a correlation distance of 40 mm in the longitudinal direction for 3000 gauss field in a positive column. Chen

and Cooper [13] have shown correlation distances of inches in a reflex discharge with one hot and one cold cathode. It is possible for the mode of operation employed with the reflex arc, that the correlation distance might have been exceeded. Another means which was tried to bring the signal through the noise was to move the 2 coils together and encapsulated in boron nitride as shown in Fig. 5. In this case the signal was received with little noise interference. However, there was no absorption or change in received signal. This was probably due to the strong coupling thru the center of the cores where there was no plasma present (Fig. 5) and possibly only a slight undetectable absorption in the plasma.

5. Results and Conclusions

Although response curves were obtained in some cases, it was not possible to duplicate the results. In no case was a resonance obtained. It is felt that if a means can be obtained to successfully reject this noise and with certain equipment improvements it will be possible to detect the ion-ion hybrid resonance or at least observe an increased absorption in the neighborhood of the expected resonance as reported by Kristiansen, et al. [14,15,16] in their investigations using other methods. These investigators found, using diamagnetic and magnetic probes, that the absorption curve shows a maximum near the hybrid frequency. However, they were unable to find a clear resonance effect.

6. Recommendations for Further Work

It is recommended that the investigation of the ion-ion hybrid resonance be continued in the reflex arc plasma machine. However, some improvements to equipment and methods are also recommended.

First, the problem of noise must be remedied. Initially, it is recommended that the P A R lock-in amplifier be rechecked before using again. It is felt that this instrument is capable of adequate noise rejection when properly adjusted.

If this does not solve the noise problem and no other adequate filter mechanism is available, it is thought that a third coil and differential amplifier system might work with some modifications. First, the correlation distance should be checked using the method of Woehler [12] or Chen and Cooper [13]. Then the coils should be constructed to fall within this distance.

Also, the coils should be close enough and small enough to be in a region of constant density. It is felt that this is necessary, since, as Buchsbaum has stated, the relative ion concentrations may vary with density. If this is the case and there is an appreciable density gradient, the response instead of appearing as a resonance curve or a sharp absorption peak would appear as an increase in absorption. This would have a maximum at the point where the density has a relative concentration for the ion species involved which produces an optimum resonance. Also, in relation to the coils it is recommended that one coil be mounted so that it will be possible to change the spacing between it and the other coil.

It is next suggested that the basic assumption be verified, i.e. the ion-ion hybrid resonance is not a function of the direction of propagation for two inductively coupled coils. This could be accomplished by following the procedures used in references 17, 18 and 19. Some modifications to the assumptions of these procedures would be necessary, however. If the basic assumption is shown to be theoretically

not valid, then the coil axis should be accurately oriented perpendicular to the magnetic field. The reason for the necessity to orient the coils perpendicular to the field can be seen from Fig. 7, which shows the variation of the resonance frequency with a change of angle from 90°. (See Appendix II.) Reshotko [6] showed how the resonance frequency varies for 50% H_1^+ and 50% H_2^+ plasma (Fig. 3). Fig. 7 shows that for the mixtures used in these investigations the change in frequency is more rapid than that reported by Reshotko. In fact the variation increases with increased mass of the ions, with decreased M_1/M_2 ratio where $M_2 > M_1$, and with decreased concentration of the lighter ion.

These facts can be seen by taking Reshotko's equation for resonance at arbitrary θ

$$\omega^2 = \frac{\Omega_1^2 + \dfrac{\omega_{ii}^2 \omega_{ei}^2}{\Omega_e^2} \tan^2\theta}{1 + \dfrac{\omega_{ei}^2}{\Omega_e^2} \tan^2\theta}$$

and putting it in the form

$$\omega^2 = \frac{\dfrac{\Omega_1^2}{\tan^2\theta} + \dfrac{\omega_{ii}^2 \omega_{ei}^2}{\Omega_e^2}}{\dfrac{1}{\tan^2\theta} + \dfrac{\omega_{ei}^2}{\Omega_e^2}}$$

Now for a krypton-neon mixture this becomes

$$\frac{\omega^2}{\Omega_1^2} = \frac{\dfrac{1}{\tan^2\theta} + \dfrac{(X_1/4 + X_2)}{1.6 \times 10^5}}{\dfrac{1}{\tan^2\theta} + \dfrac{(X_1 + X_2/4)}{4 \times 10^4}}$$

If $\theta = 89°$ AND $X_1 = X_2 = .5$

then $\omega = .98\,\Omega_1$.

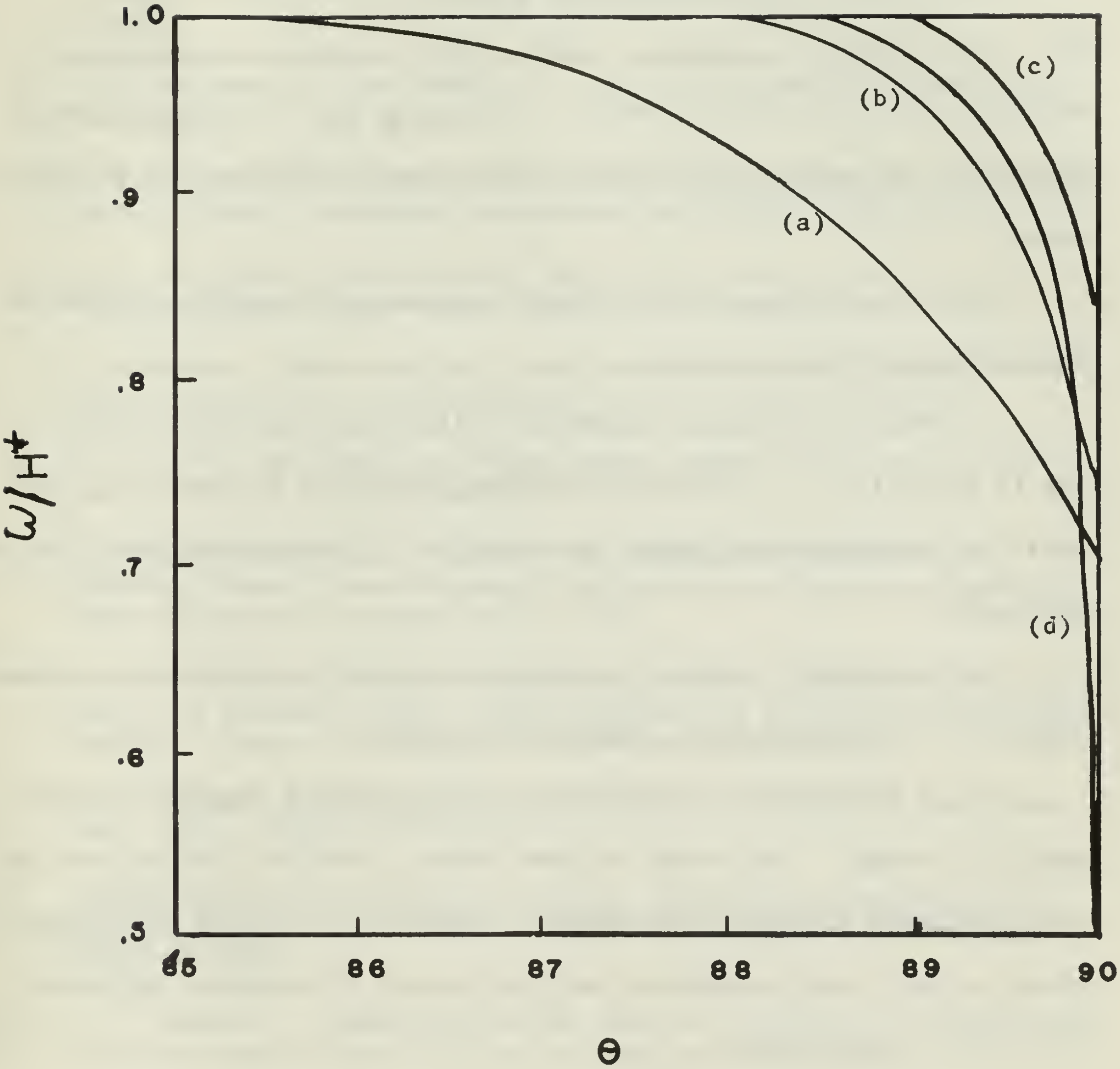


Fig. 7 Resonant frequency for various two ion species plasmas

(a) 50 percent H - 50 percent H_2
(b) 50 percent Ne - 50 percent Kr
(c) 25 percent Ne - 75 percent Kr
(d) 50 percent Ne - 50 percent A

Note. Frequency is normalized by the ion cyclotron frequency, H^+, of the lighter ion species.

If $\theta = 89°, X_1 = .25$ AND $X_2 = .75$

(this is the mixture used in most of this investigation), then

$$\omega = .99\ \Omega_1 .$$

The ion-ion hybrid resonance, however, for transverse propagation is expected at $\omega_{ii} = .5\ \Omega_1$, and $\omega_{ii} = .68\ \Omega_1$, respectively. Therefore, the acute criticality of the probe orientation can be easily seen.

Now, assuming the ion-ion hybrid resonance is detected the following recommendations are made.

(1) Reshotko's theory concerning orientation should be checked to see if it applies to inductively coupled coils. If it does apply the shift of resonance with change of direction of propagation should be investigated.

(2) Buchsbaum's theories regarding resonance amplitude and optimum relative ion concentrations should be verified.

(3) An independent verification of the relative density of ions should be sought. This might be done using a computer program such as that available at NASA, Ames Research Center, which gives ion density based on the plasma parameters and the density of neutrals introduced into the plasma machine.

(4) When the present pyrex column is replaced with a stainless steel tube it would be convenient to check Swanson's theory of critical density. It is then intended to modify the plasma machine and use a double cathode arrangement. Possibly with this arrangement the higher density of Swanson's theory will be obtained.

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APPENDIX I

Discussion of Buchsbaum's Equation

It can be seen from Fig. 8, which is a plot of Buchsbaum's equation for the ion-ion hybrid resonance frequency for the ion-ion hybrid resonance frequency for a krypton-neon mixture in terms of the percent of krypton, that the resonance frequency tended to the cyclotron frequency of neon as the amount of krypton was reduced. This fact did not seem logical and caused some concern. The equations for this resonance as derived in references 1, 5, and 6 all lead to this result. However, when the condition of reference 4 for the ion-ion hybrid resonance is converted as follows, the resonance frequency tends to the cyclotron frequency of the ion species which is most predominant. In reference 3 the condition for this resonance is

$$\beta_s \beta_r = 1$$

where

$$\beta_r = X_1 \beta_1 + X_2 \beta_2$$

$$\frac{1}{\beta_s} = \frac{X_1}{\beta_1} + \frac{X_2}{\beta_2}$$

and

$$\beta_i = \frac{e B_o}{M_i \omega_{ii}}$$

Now if these values are substituted into the original condition and the equation is solved for ω_{ii} the result is

$$\omega_{ii}^2 = \frac{X_1 f_1 + X_2 f_2}{X_1 f_2 + X_2 f_1} \Omega_1 \Omega_2$$

as compared to Buchsbaum's equation

$$\omega_{ii}^2 = \frac{X_1 f_2 + X_2 f_1}{X_1 f_1 + X_2 f_2} \Omega_1 \Omega_2$$

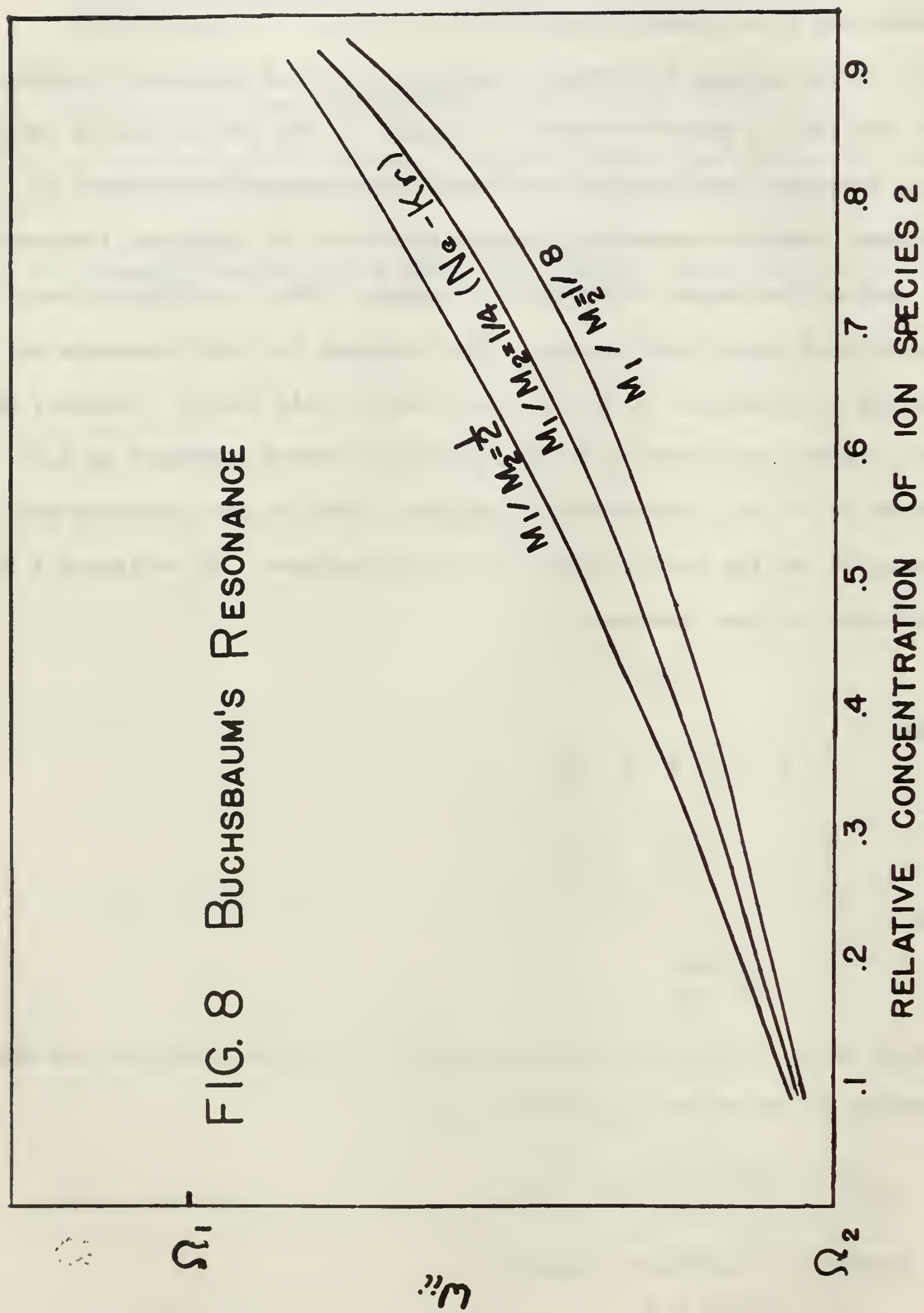

FIG. 8 BUCHSBAUM'S RESONANCE

This then left at least one equation which was incorrect. Reference 3 states "that the larger the relative concentration of a particular ion species the closer does its hybrid frequency approach the cyclotron frequency of the next lighter ion." In view of this, its experimental verification [3], and the independent derivations of references 5 and 6, it is concluded that reference 4 is in error and should be corrected as follows:

$$\beta_r = X_1\beta_2 + X_2\beta_1$$

$$\frac{1}{\beta_s} = \frac{X_1}{\beta_2} + \frac{X_2}{\beta_1}$$

SINCE

$$\omega_{ii}^{2} = \frac{X_1 M_1 + X_2 M_2}{X_1 M_2 + X_2 M_1}\,\Omega_1 \Omega_2$$

OR

$$1 = \frac{X_1 M_1 + X_2 M_2}{X_1 M_2 + X_2 M_1}\,\beta_1 \beta_2 \, .$$

ALSO

$$\beta_s \beta_r = 1$$

$$\therefore \quad \beta_s \beta_r = \frac{X_2\beta_1 + X_1\beta_2}{X_1\beta_1 \quad X_2\beta_2}\,\beta_1 \beta_2 \, .$$

APPENDIX II

Source of Fig. 7

When Reshotko's equation [6] is put in the form

$$\omega^2 = \frac{\dfrac{\Omega_1^2}{\tan^2\theta} + \dfrac{\omega_{ii}^2\omega_{ei}^2}{\Omega_e^2}}{\dfrac{1}{\tan^2\theta} + \dfrac{\omega_{ei}^2}{\Omega_e^2}},$$

and then Ω_2 is taken as a function of Ω_1 , we get an equation like the following which is for a krypton-neon mixture with the subscript 1 applying to neon, the lighter of the two ion components.

$$\frac{\omega^2}{\Omega_1^2} = \frac{\dfrac{1}{\tan^2\theta} + \dfrac{\Omega_1}{4\Omega_e}\left(\dfrac{X_1}{4} + X_2\right)}{\dfrac{\Omega_1}{\Omega_e}\left(X_1 + \dfrac{X_2}{4}\right)}$$

Substitution into this equation for various mixtures and substitution into similar equations for neon-argon and H^+ - H_2^+ mixtures leads to the following table, which is plotted in Fig. 7.

θ	ω for 50% Ne 50% Kr	ω for 25% Ne 75% Kr	ω for 50% Ne 50% Ar	ω for 50% H^+ 50% H_2^+
90°	$.5\ \Omega_1$	$.681\Omega_1$	$.707\Omega_1$	$.707\Omega_1$
89.5°	$.934\Omega_1$	$.966\Omega_1$	----	----
89°	$.981\Omega_1$	$.990\Omega_1$	$.972\Omega_1$	$.849\Omega_1$
87°	$.998\Omega_1$	$.999\Omega_1$	$.996\Omega_1$	$.981\Omega_1$

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13. ABSTRACT

The ion-ion hybrid resonance was investigated in neon-krypton and neon-argon plasmas. A signal was transmitted from one coil to a second coil to which the first was inductively coupled. A strong noise interference was encountered. A lock-in amplifier and filter system was tested for noise rejection, as was a system employing a third coil and a differential amplifier circuit. Variation of power transmitted and varying coil spacing were also tried. Some results were obtained but they were not definite or reproducible since the noise rejection techniques were not adequate. A review of the various theories pertaining to the ion-ion hybrid resonance and recommendations for further work are presented.

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